REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION	1b. RESTRICTIVE MARKINGS
Unclassified	N/A

DTIC ELECTE D
1 6 1989

AD-A205 448

2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT
	Approved for public release; distribution unlimited
2b. ... ER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)
	N/A

6a. NAME OF PERFORMING ORGANIZATION	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION
Defense Mapping Agency	PA	DMA Systems Center

6c. ADDRESS (City, State, and ZIP Code)	7b. ADDRESS (City, State, and ZIP Code)
Building 56, U.S. Naval Observatory Washington, D.C. 20305-3000	2301 Greensboro Drive, Suite 800 McLean, VA 22102

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
DMA Systems Center	EGS	N/A

8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS			
2301 Greensboro Drive, Suite 800 McLean, VA 22102	PROGRAM ELEMENT NO.	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO.
	N/A	N/A	N/A	N/A

11. TITLE (Include Security Classification)
Problems Encountered in the Stokes' Numerical Integration of Terrestrial Gravity Anomalies.

12. PERSONAL AUTHOR(S)
Weigel, Gary R.

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Technical Paper	FROM N/A TO ____	1989 January 18	43

16. SUPPLEMENTARY NOTATION

Prepared for Bulletin Geodesique/Defense Mapping Agency Technical Report.

17.	COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	World Geodetic System 1984 Stokes' Integral
			Ellipsoid Height Geopotential Coefficients
			Geoid Undulation Terrestrial Gravity Anomaly

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Various sets of gravimetric geoid undulations were computed at 600 globally distributed Doppler station sites using geopotential coefficients and different cap sizes of terrestrial gravity data. The means and standard deviations of the residuals between these gravimetric undulations and corresponding Doppler-derived geometric undulations were then computed for each set and compared. The gravimetric undulations were determined using either the OSU 81 or WGS 84 set of geopotential coefficients, each complete to degree and order 180, or a combination of the OSU 81 model truncated to degree and order 36 and terrestrial gravity data consisting of a global set of $1° \times 1°$ mean free air anomalies. The radii of the caps containing the terrestrial data were, by design, varied from $0°$ to $65°$. The Doppler undulations were derived by subtracting the mean sea level elevations at the stations from the Doppler-determined ellipsoid heights. The smallest standard deviations of the geoid undulation residuals were obtained when using a cap size of $6°$. Unexpectedly, as the cap size was increased from $6°$, increasingly larger standard deviations were

(back)

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code)	22c. OFFICE SYMBOL
Gary R. Weigel	202-227-2152	EGS

DD Form 1473, JUN 86   Previous editions are obsolete.   SECURITY CLASSIFICATION OF THIS PAGE

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Problems Encountered in the Stokes' Numerical Integration of Terrestrial Gravity Anomalies

By
Gary Weigel

Defense Mapping Agency
Washington, D.C. 20305-3000

Abstract

Various sets of gravimetric geoid undulations were computed at 600 globally distributed Doppler station sites using geopotential coefficients and different cap sizes of terrestrial gravity data. The means and standard deviations of the residuals between these gravimetric undulations and corresponding Doppler-derived geometric undulations were then computed for each set and compared. The gravimetric undulations were determined using either the OSU 81 or WGS 84 set of geopotential coefficients, each complete to degree and order 180, or a combination of the OSU 81 model truncated to degree and order 36 and terrestrial gravity data consisting of a global set of $1^0 \times 1^0$ mean free air anomalies. The radii of the caps containing the terrestrial data were, by design, varied from 0^0 to 65^0. The Doppler undulations were derived by subtracting the mean sea level elevations at the stations from the Doppler-determined ellipsoid heights. The smallest standard deviations of the geoid undulation residuals were obtained when using a cap size of 6^0. Unexpectedly, as the cap size was increased from 6^0, increasingly larger standard deviations were produced. This basic trend remained after the global set was divided into eastern and western hemispheres. Studies of the change in gravimetric geoid undulation accuracies with increasing cap size for fifteen regional areas were also made with varying results. A literature search uncovered two key factors which helped to explain this decrease in accuracy with increasing cap size found for the inter-regional solutions. It was found that 1) propagation error in the Stokes' numerical integration of erroneous mean terrestrial gravity anomalies increases rapidly with increasing cap size and 2) significant errors can result from using terrestrial gravity data which have been referenced to inconsistent vertical datums.

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1. INTRODUCTION

In 1975, *Rapp and Rummel* published a report entitled "Methods for the Computation of Detailed Geoids and their Accuracy." It included two different models for computing geoid undulations using geopotential coefficients and terrestrial $1^0 \times 1^0$ mean free air gravity anomalies. Both methods gave nearly the same result, producing a standard deviation of 0.09 m in their residuals. However, it was found that Molodenskii's method was amenable to a more simplified error analysis of the geoid undulation accuracy.

This error analysis consisted of estimating gravity anomaly data errors, geopotential coefficient errors, and the effect of truncating the geopotential coefficients at a finite degree and order. The calibration area used in their study ranged from 20^0 to 40^0 north latitude and from 277^0 to 297^0 east longitude.

Based upon the gravity data and associated error estimates available, the error analysis indicated that using larger cap sizes should produce more accurate geoid undulations than would the application of smaller cap sizes. This is certainly what one would expect since the terrestrial gravity data used within the cap contain more higher frequency gravity field information than the gravity data used outside the cap. Computations were made for caps of radii 10^0 and 20^0, using $1^0 \times 1^0$ mean anomalies within the cap and GEM 6 (*Lerch et al.*, 1974) geopotential coefficients truncated to degree 16 outside the cap. Results were then compared with undulations computed by *Marsh and Vincent* (1973) from Geos-3 altimeter data. This comparison apparently confirmed their error estimation theory to within the expected noise level: mean differences of 4.15 m and 3.87 m and standard deviations of 3.52 m and 2.58 m were obtained for 10^0 and 20^0 cap sizes, respectively.

In part, our present study provides a check on the implicit assumption of *Rapp and Rummel's* error analysis that the geoid undulation accuracy should generally increase with increasing cap size. We also used Molodenskii's method (cf. Section 2) for computing gravimetric geoid undulations. Our "control" data consisted of a global set of 600 Doppler-derived geometric geoid undulations, each computed at the station's survey marker through subtracting the station's mean sea level elevation, taken as an approximation of orthometric height, from its Doppler-determined ellipsoid height. All undulations were referenced to the World Geodetic System 1984 (WGS 84) (*DMA Report,* 1987).

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This set of Doppler undulations was then compared with various sets of gravimetric undulations covering the cap range of 0^0 to 65^0, where terrestrial $1^0 \times 1^0$ mean free air gravity anomalies were used within the cap. The OSU 81 (*Rapp,* 1981) Earth Gravity Model (EGM) truncated to degree and order 36 was used to compute the undulation component outside the cap. Two additional sets of gravimetric undulations were computed using the complete 180 degree and order OSU 81 and WGS 84 EGM's.

Beyond a cap of 6^0 radius, it was found that the standard deviations of the geoid undulation residuals (gravimetric minus Doppler) steadily increased as the cap size, within which terrestrial data was used, was increased. This basic trend remained after the global station set was divided into eastern and western hemispheres. However, when the global set of 600 stations was divided into fifteen regional areas, for most regions this same increase in magnitude was not reproduced, although there was some increase in the standard deviations of the solutions for several of the regions.

Section 5 will be devoted to analyzing and proposing possible explanations for this increasing "instability" in the solutions with increasing cap size. We will find that when using Stokes' method with mean terrestrial gravity anomalies, errors propagate very rapidly with increasing cap size (*Despotakis,* 1987). Following the lead given by *Rapp* (1983) and *Laskowski* (1983), we will also discover that the effect is to a significant extent the result of our using gravity anomalies which have been referenced to inconsistent vertical datums.

2. MATHEMATICAL MODEL

The mathematical model used for computing the gravimetric geoid undulation N_G was based on the theory presented by *Rapp and Rummel* (1975). In their report, N_G consisted of three components: (1) the N_1 component was computed outside a cap of specified radius about the computation point using an EGM; (2) the N_2 component was computed inside the cap using Stokes' integral and mean free air gravity anomalies; and (3) N_3 was the component of the geoid undulation which represented the effect of truncating the EGM at a finite degree and order. (Note: Primes were used on N_1, N_2, and N_3 by *Rapp and Rummel* to distinguish their *Method B* from their *Method A.* They will be omitted here.)

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Thus the model for computing the gravimetric undulation was

$$N_G = N_1 + N_2 + N_3 \tag{1}$$

where:

$$N_1 = \frac{R}{2G} \sum_{l=2}^{l_{max}} Q_l(\Psi_0) \, \Delta g_l \tag{2}$$

$$N_2 = \frac{R}{4\pi G} \iint_{\sigma_c} \overline{\Delta g}^0 \, S(\Psi) \, d\sigma \tag{3}$$

$$N_3 = \frac{R}{2G} \sum_{l=l_{max}+1}^{\infty} Q_l(\Psi_0) \, \Delta g_l \tag{4}$$

and where:

$$Q_l(\Psi_0) = \int_{\Psi_0}^{\pi} S(\cos\Psi) \, P_l(\cos\Psi) \sin\Psi \, d\Psi$$

$$\Delta g_l = \frac{GM}{r^2} (l-1)\left(\frac{a}{r}\right)^l \sum_{m=0}^{l} (\overline{C}_{lm}^* \cos m\lambda + \overline{S}_{lm} \sin m\lambda) \, \overline{P}_{lm}(\sin\overline{\phi})$$

R ..... Mean earth radius

G ..... Mean value of gravity over the entire earth

l_{max} ..... Maximum degree of geopotential coefficients

$Q_l(\Psi_0)$ ..... Molodensky truncation function

Ψ_0 ..... Cap size of gravity to be included in N_2

$\overline{P}_{lm}(\sin\overline{\phi})$ ..... Normalized Legendre polynomial of degree l
and order m for the geocentric latitude $\overline{\phi}$

$\overline{\Delta g}^0$ ..... Approximate mean free air gravity anomaly

σ_c ..... Cap of radius Ψ_0

$P_l(\cos\psi)$ ..... l th degree Legendre polynomial

Δg_l ..... l th degree component of the gravity anomaly implied by a set of geopotential coefficients

GM ..... Geocentric gravitational constant

r ..... Geocentric distance to the point of computation

a ..... Equatorial radius

$\bar{C}^*_{lm}, \bar{S}_{lm}$ ..... Normalized differences between the actual geopotential coefficients and those implied by the adopted reference ellipsoid

The Doppler-derived undulation was computed from the equation

$$N_D = h - H, \qquad (5)$$

where:

h ..... Doppler-determined ellipsoid height

H ..... Mean sea level elevation (taken as orthometric height).

After obtaining values for N_G and N_D at each Doppler site, the least squares adjustment was computed based upon the following observation equation:

$$N_G - N_D = \cos\phi\cos\lambda\,\Delta X + \cos\phi\sin\lambda\,\Delta Y + \sin\phi\,\Delta Z - N_0 \qquad (6)$$

where the observations consisted of

ϕ ..... Geodetic latitude

λ ..... Geodetic longitude

N_G .... Gravimetric geoid undulation

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N_D ..... Doppler-derived geometric geoid undulation.

The unknown parameters solved for in the adjustment were

ΔX, ΔY, and ΔZ ..... Doppler coordinate system origin with respect to the geocenter as defined by the global gravimetric geoid.

N_0 ..... Zero order geoid undulation, which is a constant to be added to the gravimetric geoid undulations in order to reference them to the prescribed ellipsoid.

Supporting statistics obtained included the mean of the geoid undulation residuals $N_G - N_L$, the standard deviations of the residuals before and after the adjustment, σ_N and σ_V, and the standard deviation of the mean of $N_G - N_D$.

3. COMPUTATIONS

The computer program used to compute $N_1 + N_2$ was obtained from Ohio State University, Department of Geodetic Science and Surveying, Columbus, Ohio (*Wichiencharoen*, 1982), as was the least squares adjustment program. The program used to compute N_1 only, using either the complete 180 degree and order OSU 81 or WGS 84 EGM, was obtained from the National Geodetic Survey, Rockville, Maryland (*Tscherning and Goad*, 1981).

The terrestrial gravity data used in the experiments consisted of a set of $1^0 \times 1^0$ mean free air gravity anomalies obtained from the Defense Mapping Agency Aerospace Center, St. Louis, Missouri.

All gravimetric geoid undulation computations included corrections for ellipticity. Since terrestrial gravity data is not available for some areas of the world, a small percentage of the global set of gravity anomalies were computed from geopotential coefficients. These anomalies were assigned a relatively small weight in the computations. An atmospheric correction was applied to each of the $1^0 \times 1^0$ mean free air

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gravity anomalies, computed using a quadratic polynomial fit with the corresponding $1^0 \times 1^0$ mean elevation serving as the independent variable (cf. *Wichiencharoen,* 1982).

A scale change of -0.6 ppm, a rotation about the Z-axis of 0.814 arc sec, and a shift of 4.5 m in the Z-coordinate were applied to the coordinates of all Doppler stations in order to reference them to the geocentric WGS 84 coordinate reference frame.

An original set of 2153 Doppler (transformed to WGS 84) stations was then reduced to a set of 638 stations having mean sea level elevations of 100 m or less. This reduction was accomplished in order to eliminate the possible effects of systematic errors in levelling with respect to elevation.

This set of 638 stations was further reduced to a set of 600 stations through deleting 38 stations located near areas where terrestrial gravity data of questionable quality was thought to exist. The criterion for this reduction was primarily based upon juxtiposing the neighboring terrestrial anomalies with each other and with corresponding $1^0 \times 1^0$ mean gravity anomalies computed from the OSU 81 EGM. Asia, the Indian Ocean area, and Africa were the regions affected by this elimination of stations. *Pavlis* (1988) has confirmed that "terrestrial data in that region [Africa, the Soviet Union, and China] are substantially in error."

The possible inadvertent elimination of some good data should not detract from our objective; and we do not suggest that we were able to delete all of the low quality data, either. However, the isolation and elimination of whatever questionable data we could find should enhance our study through enabling us to focus on those characteristic results which can *generally* be expected to occur for moderate to high quality terrestrial gravity measurements and reductions. Then, through analysis, an attempt can be made to delineate and approximately quantify commonly occurring causes for any problematic results.

As prescribed by the mathematical model given in Section 2, the N_2 component of the geoid undulation was included in the solution using a set of terrestrial $1^0 \times 1^0$ mean free air anomalies within the cap. Outside the cap, the N_1 component was computed using the OSU 81 EGM truncated to

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degree and order 36. Results of comparisons with the global set of 600 Doppler-derived geoid undulations are given in Figures 1 through 3. A cap of size 0^0 indicates that only the N_1 component was used in the solution. The largest cap size used had a radius of 65^0, due to computer storage limitations; but the gravimetric undulations do not change substantially with respect to changing cap size for the larger caps.

The global set of Doppler stations was then divided into fifteen regions and separate solutions were obtained for each. Results were obtained for (1) the OSU 81 EGM complete to degree and order 180 (cf. Tables 1 and 2) and (2) for various cap sizes, ranging from 0^0 to 65^0 (Figures 4 through 18 and Table 1). Exactly the same method used for the global case was used here as well, i.e., $1^0 \times 1^0$ mean free air gravity anomalies were used to compute the N_2 component within the cap and the OSU 81 EGM, truncated to degree and order 36, was used to compute the N_1 component outside the cap.

Two additional sets of solutions were made for the eastern and western hemispheres (Figures 19 and 20, and Table 1).

In order to test for possible errors in the algorithms, an experiment was conducted to compare the accuracy of the Stokes' algorithm with that of the algorithm which uses geopotential coefficients only (Table 2). Separate experiments were made for the complete OSU 81 EGM and the complete WGS 84 EGM. In these experiments, $1^0 \times 1^0$ mean free air gravity anomalies were first computed entirely from the complete 180×180 EGM. These anomalies were then used in Stokes' integral to compute the N_2 component of the gravimetric geoid undulations for a cap size of 65^0 at all 600 Doppler sites. Outside the cap, the N_1 component was computed from the same EGM, but truncated to degree and order 36 due to the computational limitations of the computer program. (Factorials of the Legendre polynomials beyond a relatively low degree and order are too large in magnitude for computers; for the present experiment, however, the effect of this truncation should be very small).

Next, the gravimetric geoid undulations for all 600 Doppler sites were computed using the complete set of geopotential coefficients (i.e., Stokes' integral was not used: cap size = 0^0).

Therefore, the two sets of gravimetric geoid undulations (cap sizes of 0^C and 65^0) were computed using data generated by the EGM only (i.e., the terrestrial gravity data was not used). Any differences in the adjusted parameters or standard deviations of the geoid undulation residuals could then be attributed to differences between the two methods in their computational error propagation. Results are given in Table 2.

4. RESULTS

Solutions were first obtained for the global set of 600 Doppler stations using only the OSU 81 EGM truncated to degree and order 36 (i.e., cap size = 0^0). Adding a 1^0 cap of terrestrial data to the solution, that is, computing N_2 inside the cap using $1^0 \times 1^0$ mean free air gravity anomalies, and N_1 outside the cap using the 36 x 36 OSU 81 EGM, produced reductions of 0.65 m in the standard deviation σ_N of the residuals $N_G - N_D$ and 0.64 m in the standard deviation σ_V of the adjusted residuals (Figure 1). The smallest σ_N obtained for any solution, including the ones obtained when using the complete OSU 81 EGM only (2.45 m) or the complete WGS 84 EGM only (2.41 m), was produced when using a cap size of 6^0 (2.24 m). As the cap size was increased from 6^0, both σ_N and σ_V steadily increased as did the magnitudes of the values for the adjusted parameters ΔX and ΔY, ostensibly indicating the existence of low quality gravity anomalies in the data set (Figure 2). At the largest cap size of 65^0, these values were 2.91 m, 2.64 m, 2.18 m, and 1.95 m, respectively. The difference $\sigma_N - \sigma_V$ also increased with increasing cap size, with the smallest difference of 0.02 m occurring for the 1^0 cap size solution, and the largest difference of 0.27 m occurring for the 65^0 cap size case.

Figure 3 gives the maximum and minimum of the residuals $N_G - N_D$ for the various cap sizes. Residuals of greatest magnitude were obtained for the 0^0 cap size case where the OSU 81 EGM, truncated to degree and order 36, alone was used. Maximum and minimum residuals of smallest magnitude were obtained for the smaller cap sizes (1^0 to 10^0) and the magnitudes of the maximum and minimum residuals gradually increased with increasing cap size.

Figures 4 through 18 give results for fifteen regional areas. Smallest values for σ_N and σ_V were obtained for the east and west coasts of U.S.-Canada, the U.S. Gulf States, Europe, and the west coast of South America (although the last station set consisted primarily of a dense group of stations located in islands several arc degrees from the coast of South America; we use the term "coast" in the very broad sense). Largest standard deviations of the geoid undulation residuals were obtained for the Arctic, the east coast of South America, and the West Indies. For Europe and Australia, σ_V remained relatively small and constant while σ_N increased dramatically as the cap size was increased. It is to be noted that the latter two regions are near the area where low quality terrestrial gravity anomalies are known to exist (i.e., Africa, the Indian Ocean, and Asia; cf. Section 3, above).

Because of the existence of inaccurate anomalies in this region, separate solutions were made for the eastern and western hemispheres (cf. Figures 19 and 20). The results given for the Western Hemisphere follow the same trend of results found for the global solutions. The Eastern Hemisphere results uncharacteristically show a larger difference $\sigma_N - \sigma_V$ for the smaller cap sizes, but after 15^0 produce the same trend of increasing standard deviations with increasing cap size as found for the Western Hemisphere and global solutions.

Table 1 gives the means of the residuals $N_G - N_D$, the standard deviations σ_N and σ_V and standard deviations of the means of the residuals $N_G - N_D$ for both the case where the complete 180 degree and order OSU 81 model was used, and for the case where terrestrial gravity data was used within a cap of 65^0. Using the complete OSU 81 EGM, the range of the mean residuals over the fifteen regions was 6 m; for the 65^0 cap size solution it was 7 m. The standard deviations σ_N and σ_V and the standard deviation of the mean of the residuals $N_G - N_D$ also varied significantly from region to region.

Differences in the means of the geoid undulation residuals between the OSU 81 and 65^0 cap size solutions for any given region were generally very significant with a maximum difference of nearly 5 m occurring for the North Pacific Ocean area. Standard deviation comparisons between the

two types of solutions for the various regions generally showed significant differences in magnitude for σ_N (Australia's σ_N increased from 1.21 to 3.52 m after adding the terrestrial data) but not as much change, in most cases, for σ_V. Corresponding to what we observed earlier from the global 600 Doppler station set solutions, the difference $\sigma_N - \sigma_V$ was generally greater for the 65^0 cap size case than for the OSU 81 EGM solutions, with the only exceptions being the Atlantic Ocean area, the South Pacific, and the U.S. Gulf States.

Table 2 gives the results of the experiment conducted to compare the accuracy of the Stokes' algorithm with that of the algorithm which uses geopotential coefficients only. Here, two sets of gravimetric geoid undulations (cap sizes of 0^0 and 65^0) were computed using only the 180 x 180 EGM (i.e., no terrestrial gravity data was used). For the OSU 81 EGM, the standard deviation σ_N increased from 2.45 m for the 0^0 cap size solution to 2.50 m for the 65^0 cap size solution. For the WGS 84 EGM, σ_N increased from 2.41 m to 2.47 m. Differences in the adjusted parameters ΔX, ΔY, ΔZ, and N_0 can be rounded off to one decimeter. Since the N_1 component was computed outside the 65^0 cap using the EGM truncated to degree and order 36, we might expect to see a very small increase in the standard deviation. However, the 0.05 to 0.06 m increase we obtained can only be partially accounted for by this truncation and, as we will now proceed to explain in the following section, can also be attributed in part to the nature of our computational proceedures.

5. Analysis of Results

Statistical theory defines σ_N as the total standard error of the geoid undulation residuals; the standard deviation σ_V then gives an estimate of the precision, or remaining random error, after "systematic" effects have been removed through the least squares adjustment. Here the term "systematic" must be qualified, as some systematic effects cannot be removed due to unresolved correlations and the ill conditioned structure of "real world" observations. Supposedly though, a crude estimate of the systematic error σ_S can be obtained through a propagation of variances, such that $\sigma_S^2 = \sigma_N^2 - \sigma_V^2 + 2\sigma_{SV}$, where σ_{SV} is hopefully close to zero.

Looking at the values of σ_N and σ_V given in Table 1, we see that generally σ_S values would be larger for the 65^0 cap size solutions than the OSU 81 EGM solutions, implying the existence of greater systematic error when using the terrestrial gravity set. One notable exception was the Eastern Hemisphere solutions where we suspected a preponderance of low quality terrestrial gravity data. Uncharacteristically, σ_V was 0.4 m larger for the 6^0 cap size case than for the OSU 81 solution, while σ_N was larger by only 0.04 m.

We are therefore led to ask the following questions: What traits are inherent in the nature of the terrestrial gravity data, or possibly the Stokes' numerical integration of that data, which are apparently absent in the gravity field information contained within the EGM's? Why do these traits surface so characteristically as a function of geographical distance?

As alluded to at the end of Section 4, part of the problem can be traced to the effects of truncation error in Stokes' algorithm. Because of their noncontinuous character, the $1^0 \times 1^0$ mean free air gravity anomalies used in the Stokes' integration over a cap of specified radius must actually be summed, i.e., numerically integrated. Each $1^0 \times 1^0$ block must be divided into sub-blocks, and the Stokes' function, which "weights" the anomalies, evaluated at the center point of each sub-block and meaned (*Rapp and Rummel,* 1975). For a gravity anomaly inside a radius of 2^0 from the computation point, 64 sub-blocks are used, and this density decreases as gravity anomalies further away from the computation point are incorporated into the solution. Consequently, the evaluation of these more distant gravity anomalies by Stokes' function will be less accurate than the evaluation of those located near the computation point. Gravity anomalies of greater magnitude and located within rough gravity areas will be most affected by this numerical integration. However, based upon the results given at the end of Section 4, we know that this error source does not increase σ_N by more than 0.06 m.

Another source of error arises from the fact that all gravity anomalies used in the computations were rounded to the nearest 0.1 mgal. This creates further computational errors, which accumulate in magnitude as the cap size is increased, although these errors must also be relatively small in magnitude.

A third source of error is generated by systematic biases in the Doppler-derived ellipsoid heights with respect to geographic location. This error source could be a function of a wide range of known but not as yet precisely measured factors, including, among other things, geomagnetic effects correlated with latitude, solar activity cycles, number of satellite passes, which satellites were observed, years during which satellite observations were made, whether default values or actual local meterological observations were used as input into the tropospheric refraction model, and the coordinate transformation of the Doppler data to a geocentric reference frame.

The effects of other systematic biases may also be prevalent in our solutions but unknown at the present time. These could include, among other things, the following: systematic errors in levelling depending upon whether trigonometric or geodetic surveys, or very inaccurate indirect methods of obtaining elevations, were made in a particular area; the predominance of geophysical data in determining mean free air gravity anomalies for some areas; the possible predominance of shorter wavelengh information in the terrestrial $1^0 \times 1^0$ mean free air gravity anomalies compared with $1^0 \times 1^0$ gravity anomaly blocks derived from a combination of satellite observations and terrestrial data; the effect of the equatorial bulge on the determination of vertical datums and gravity anomalies; and whether or not the Doppler stations were well distributed or not within a given region.

Variations in the roughness of topography upon which gravity observations are made can be another source of systematic error. It is well known that terrestrial gravity data refer to the earth's topography. This topography may significantly deviate from an equipotential surface (cf., e.g., *Cruz*, 1985). Thus, in rough topographic areas (e.g., mountainous regions) gravity anomalies cannot be adequately reduced to the ellipsoid because of the widely fluctuating shape of the topography. We must suspect that in many areas we are using in our Stokes' integration $1^0 \times 1^0$ mean free air anomalies which have been derived from point gravity anomalies obtained in rough topographic areas.

Given the complexity of our problem implied by the abundant results and provisional explanations suggested to this point, the reader may understandably be skeptical that its opaque and interwoven texture could be pervious to further enlightenment. Any reservations given to further

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analysis will hopefully be placated, however, through a brief summary of some relevant literature. It is our contention that this review will enable us to look at our problem from a fresh angle and thereby bring into relief some as yet unresolved key factors. We therefore ask the reader to bear with us as we turn our attention for the moment to 1) studies of the Stokes' numerical integration as a function of cap size presented by *Despotakis* (1987) and 2) analyses of the effects of variations in vertical datums made by *Rapp* (1983) and *Laskowski* (1983).

Despotakis has provided a rigorous error analysis of the Stokes' method. He considered many of the error sources already mentioned as well as some we have not yet considered. Of primary concern was his finding of an increase in the global RMS gravimetric undulation error of approximately 0.7 m from the 2^0 cap size to the 10^0 cap size solution. He attributed this result to propagation error inherent in the Stokes' method of numerical integration of erroneous mean values of terrestrial gravity anomalies.

With this instructive knowledge now in hand, our reborn optimism would have us recall our earlier suspicion concerning the character of the terrestrial data because of this same increase in the standard error of the geoid undulation residuals with respect to increasing cap size. The contradiction, left hanging, over why we had obtained such good results for the smaller cap sizes, when we strongly suspected that we were using low quality gravity anomalies – as evidenced by the larger cap size solutions – was at that time prudently "brushed under the geoid." However, the improvement of 0.67 m in the standard deviation σ_N for the 6^0 cap size solution over the 65^0 cap size solution is now made comprehensible from the study of *Despotakis* (1987) summarized above.

But how to account for the fact that the 6^0 cap size solution gave improvements of 0.21 m and 0.17 m over the standard deviations obtained when using the OSU 81 and WGS 84 180 degree and order geopotential sets, respectively? This comparison provisionally assumes, of course, that the gravity field spectral information and quality of gravity data contained in the 180 degree and order EGM's and the global set of terrestrial gravity data should be approximately the same.

In his 1983 presentation at the National Geodetic Survey, Professor *R. H. Rapp* of Ohio State University explained that due to such effects as salinity differences, temperature differences, atmospheric pressure, winds, crustal movement, eustatic changes, and river discharge, the mean sea level of any vertical datum is a function of time and is not the geoid, but only an approximation thereof. Furthermore, since generally many different tide gauge locations are used in the adjustment of any given vertical datum, its height system is basically warped. He also stated that the various vertical datums of the world are systematically inconsistent with one another at about the 1.5 m level. These errors in vertical datums would then create errors in the use of gravity data. For example, gravity anomalies after reduction would not refer to a common equipotential surface.

Laskowski presented in his revealing report the results of simulation studies using three different models of inconsistency in the world vertical datum due to variations in sea surface topography. Sea surface topography was defined as the departure of the mean sea level from the equipotential surface. Based on his distortion models, the effect of datum inconsistency can be on the order of 0.5 m for geoid undulations computed using 180 degree and order EGM's which have been created using *only terrestrial data*. However, he found that the effect of datum inconsistency is almost entirely limited to the lower degree harmonics, so that, for example, the results presented here using the 180 degree and order OSU 81 and WGS 84 EGM's should be negligibly affected by world datum inconsistencies as their lower degree harmonics were determined *primarily by satellite data*. This will certainly not be the case when using Stokes' integral with terrestrial data, however; and furthermore, with increasing cap size, the gravity anomalies which have been referenced to inconsistent equipotential surfaces will propagate increasingly larger errors in computations of the geoid undulations. *Laskowski* found this error to be about 0.3 m for a cap size of 65^0 when using his simulated distortion models.

We recall that the Doppler-derived WGS 84 geoid undulations were computed by subtracting the mean sea level elevation at the station from the Doppler-derived (transformed to WGS 84) ellipsoid height. However, we now know that gravity anomalies, as discussed by *Rapp* (1983) and *Laskowski* (1983), are referenced to a mean sea level which is not the

true geoid. We can further suppose that for sufficiently small areas about the Doppler station, such as, for example, we suspect could occur for the 6^0 cap size solution, generally the mean sea level used to compute the WGS 84 Doppler-derived geoid undulation and that used to reduce the terrestrial $1^0 \times 1^0$ mean free air anomalies are equivalent, or nearly equivalent, and refer to the same vertical datum.

Concerning solutions using small cap sizes then, it is now possible to conjecture that *discrepancies between the mean sea level and the true geoid would generate approximately the same error in both the gravimetric (terrestrial data only) and the Doppler-derived geoid undulations. Therefore, with respect to the computations of σ_N, and σ_V in the least squares adjustment, this error would be subtracted out of the solutions through the computation of the residual $N_G - N_D$*. This might then account for the smaller standard deviations we obtained for the small cap size solutions contrasted with those we obtained when using the 180 degree and order EGM's only.

Consider finally the proposition that the means of the residuals $N_G - N_D$ for the fifteen regions investigated can be used as approximations for the divergence of vertical datums (i.e., mean sea levels) from a global equipotential surface representing a "true geoid" (cf. Table 2). These estimates must be based upon the assumption, however, that global systematic error biases in the Doppler-derived ellipsoid heights with respect to geographic location are relatively small. This assumption might be valid to within the noise level of our present study, although it would be difficult to prove. However, concerning the gravimetric undulations it can only be valid for the solutions obtained when using the 180 degree and order EGM's. Based on our argument, the means of $N_G - N_D$ obtained when using terrestrial gravity data to compute the N_2 component of N_G cannot be used to estimate these vertical datum differences due to the fact that the gravity anomalies contained within a *large* cap are referenced to *inconsistent and different* vertical datums, and for sufficiently *small* cap sizes, are referenced to the *same* vertical datum as the Doppler-derived geometric geoid undulations.

6. SUMMARY

We were concerned with showing the *very general* effects of including increasingly larger quantities of terrestrial data in the solutions for gravimetric geoid undulations using Stokes' integral.

When Stokes' integral was used with the set of $1^0 \times 1^0$ mean free air gravity anomalies, the "best fitting" geoid undulations for the global solutions and the eastern and western hemisphere solutions were generally computed for the smaller cap sizes, taking Doppler-derived WGS 84 geometric geoid undulations as the "control" data. Beyond a relatively small cap size the accuracy of the gravimetric undulations steadily decreased with increasing cap size. The only exception was the 30^0 to 50^0 cap size range for the Western Hemisphere solutions. Results for the fifteen regions studied, however, often did not follow this trend and were much more variable. The fog surrounding these results was dispersed somewhat by deducing that the systematic errors inherent in the gravimetric geoid undulations, which are so characteristically a function of increasing cap size, can be attributed among other things to the effect of propagation error in the Stokes' method and to the apparently universal existence of inconsistent vertical datums to which the gravity anomalies were reduced.

A further breakdown of the reasons for the decreasing gravimetric geoid undulation accuracy (measured against the Doppler-derived undulation) as the cap size of the solution was increased can be given as (1) the effect of numerically integrating the Stokes' function computations thereby producing truncation errors, (2) the effect of the input terrestrial data – the $1^0 \times 1^0$ mean free air gravity anomalies – being rounded off to the nearest 0.1 mgal, (3) systematic errors in the Doppler-derived ellipsoid heights with respect to geographic location, (4) other unknown sytematic biases, (5) the fact that the gravity anomaly data referred to the earth's topography and therefore could not be adequately reduced to the ellipsoid in rough topographical areas, (6) the problematic existence of lower quality gravity anomalies in areas located further away from the Doppler sites, (7) propagation error in the Stokes' numerical integration of erroneous mean terrestrial gravity anomalies as a function of cap size, and (8) inconsistencies between local vertical datums within regional vertical datums, compounded with systematic differences between the various regional vertical datums of the world.

The latter effect is theoretically correctable through the application of advanced geodetic techniques (cf., e.g., *Rapp,* 1983 and 1985 and *Laskowski,* 1983). For this reason we emphasize, with *Rapp* and *Laskowski,* the urgency of establishing a consistent unified world vertical datum so that gravity anomalies and, in fact, all gravity related measurements can be reduced to a common equipotential surface. Also, methods for taking into account global topographic/isostatic effects (*Rummel et al.,* 1988) and propagation error in the Stokes' numerical integration of mean terrestrial gravity anomalies as a function of increasing cap size (*Despotakis,* 1987) have been developed and should be implemented for computations of geoid undulations from terrestrial gravity anomalies.

Acknowledgements

I wish to thank Archie Carlson and Fran Varnum for helpful discussions and Louis Decker for providing the terrestrial gravity data.

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Table 1.

Statistics for Geoid Undulation Residuals $N_G - N_D$

Region	Number of Stations	OSU 81 EGM (180 x 180)			Cap Size of 65°		
		Mean	Std Dev σ_N (σ_V) (Meters)	Std Dev of Mean	Mean	Std Dev σ_N (σ_V) (Meters)	Std Dev of Mean
Global	600	0.20	2.45 (2.43)	0.10	0.06	2.91 (2.64)	0.12
Alaska	58	0.37	2.28 (1.79)	0.30	-0.13	2.52 (1.97)	0.33
Arctic	35	-1.12	2.88 (2.34)	0.49	-1.10	3.46 (2.82)	0.59
Atlantic	17	-2.31	2.69 (1.07)	0.65	0.66	2.05 (1.10)	0.50
Australia	26	2.37	1.21 (1.14)	0.24	-0.46	3.52 (1.14)	0.69
East Coast of South America	25	0.78	2.13 (1.98)	0.43	1.27	3.98 (3.25)	0.80
East Coast of U.S.-Canada	75	0.61	1.63 (1.49)	0.19	0.51	1.69 (1.41)	0.20
Europe	71	1.27	1.25 (1.18)	0.15	2.61	2.20 (1.23)	0.26
Mexico-Central America	31	-0.34	2.40 (2.18)	0.43	-2.91	2.90 (1.96)	0.52
North Pacific	41	-3.69	2.21 (1.81)	0.35	1.26	2.26 (1.72)	0.35
Northwestern Africa	15	0.09	3.01 (2.80)	0.78	4.17	2.58 (2.10)	0.67
South Pacific	17	1.56	3.60 (2.04)	0.87	-1.22	2.79 (2.36)	0.68
U.S. Gulf States	40	1.55	0.83 (0.70)	0.13	0.04	0.70 (0.69)	0.11
West Coast of U.S.-Canada	77	1.02	1.13 (1.03)	0.13	-1.83	1.28 (0.94)	0.15
West Coast of South America	16	-0.70	0.95 (0.54)	0.24	-2.75	1.19 (0.63)	0.30
West Indies	56	-0.62	2.59 (2.42)	0.35	0.17	3.17 (2.61)	0.42
Eastern Hemisphere	201	-0.11	3.00 (2.43)	0.21	1.24	3.04 (2.83)	0.21
Western Hemisphere	399	0.36	2.11 (2.08)	0.11	-0.54	2.65 (2.48)	0.13

Table 2.

Adjusted Parameters and Standard Deviation of the Residuals $N_G - N_D$:
All gravimetric data was obtained from either the OSU 81 or WGS 84
EGM. N_G was computed 1) with and 2) without using Stokes Integral.

Model Used	ΔX (Meters)	ΔY (Meters)	ΔZ (Meters)	N_0 (Meters)	Std Dev σ_N (σ_V) (Meters)
OSU 81 $N_1 + N_2$ Cap = 65^0	0.71	0.09	-0.28	-0.31	2.50 (2.48)
N_1 Only	0.59	-0.05	-0.38	-0.36	2.45 (2.43)
WGS 84 $N_1 + N_2$ Cap = 65^0	0.59	0.01	0.09	-0.16	2.47 (2.45)
N_1 Only	0.49	-0.13	0.02	-0.19	2.41 (2.40)

FIG 1. STANDARD DEVIATIONS OF
GRAVIMETRIC – DOPPLER UNDULATION RESIDUALS FOR GLOBAL SOLUTIONS.
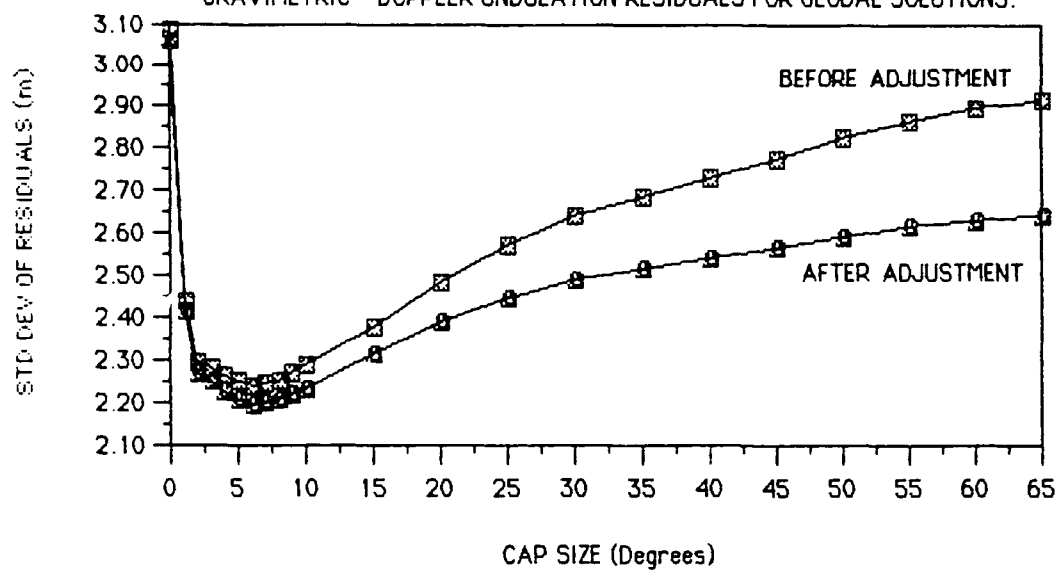
FIG 2. GRAVIMETRIC – DOPPLER PARAMETERS FOR GLOBAL SOLUTIONS AFTER APPLYING LEAST SQUARES ADJUSTMENTS.
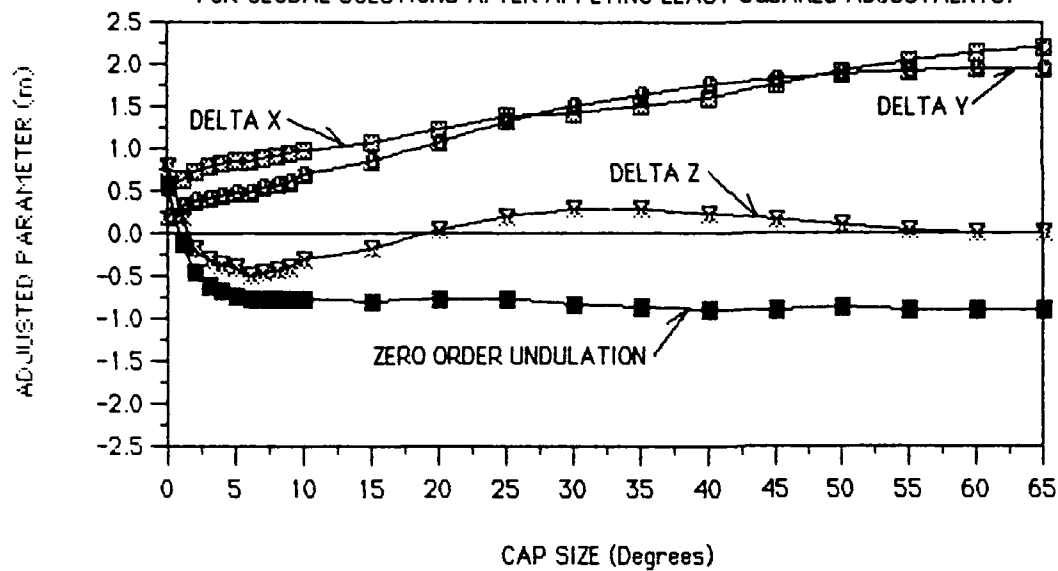
FIG 3. MAXIMUM AND MINIMUM OF
GRAVIMETRIC – DOPPLER UNDULATION RESIDUALS FOR GLOBAL SOLUTIONS.
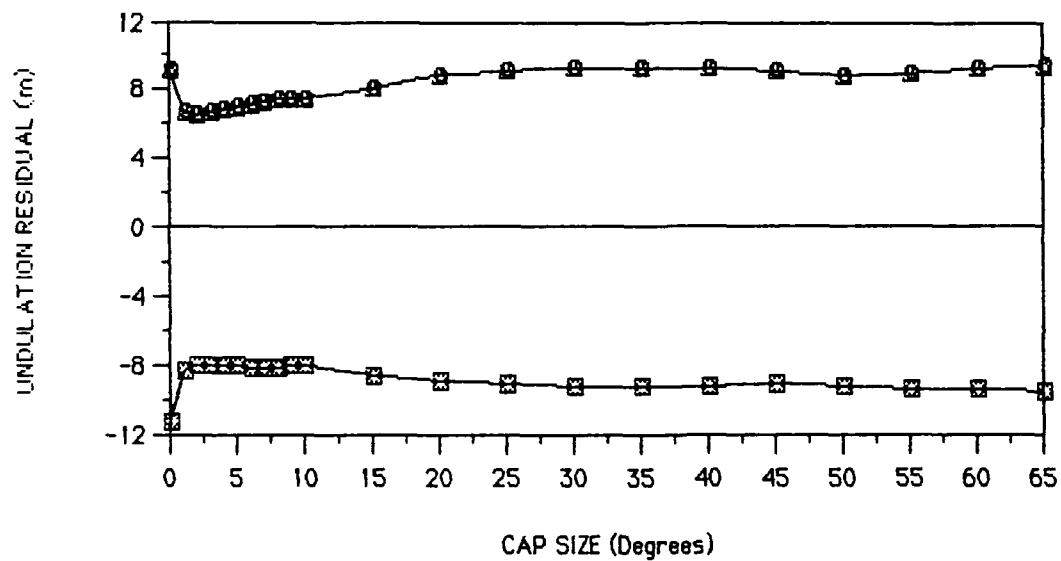
FIG 4. ALASKA.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.
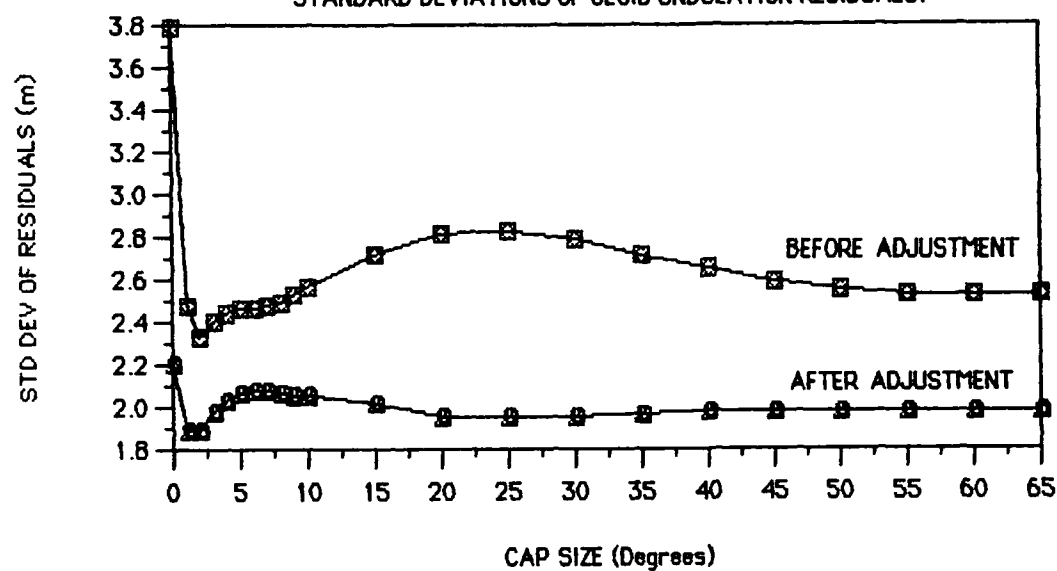
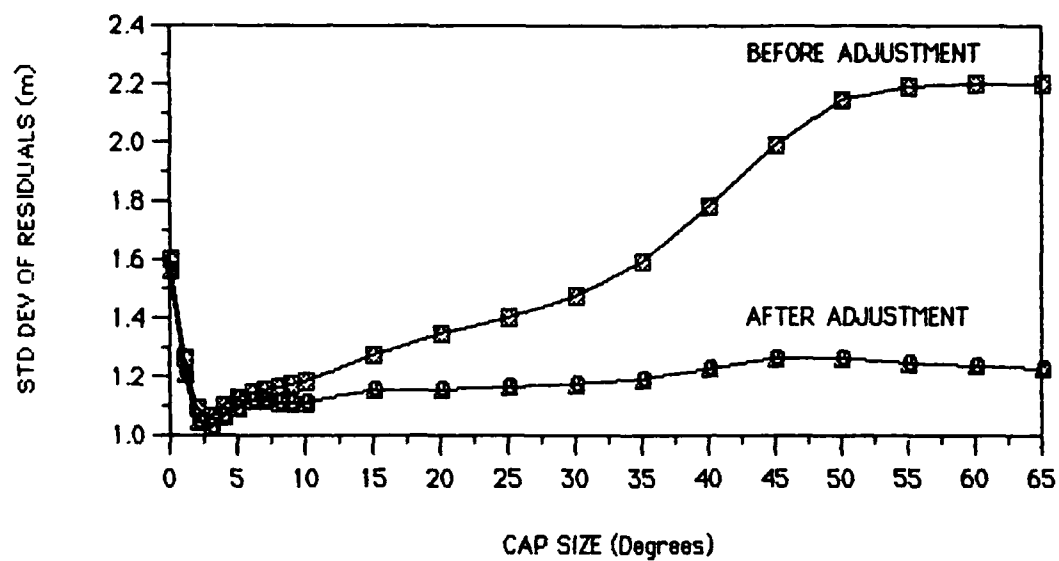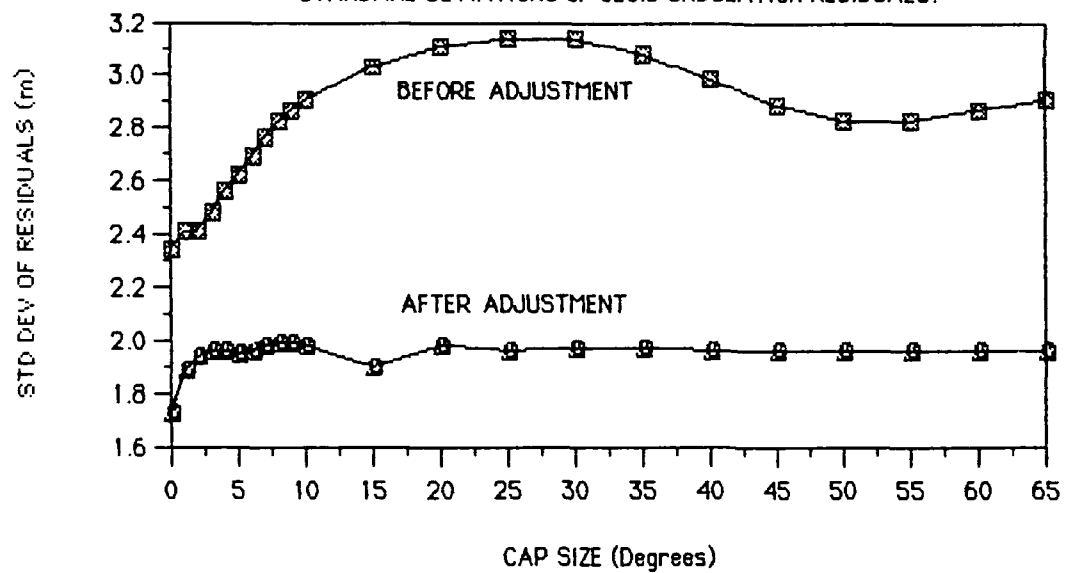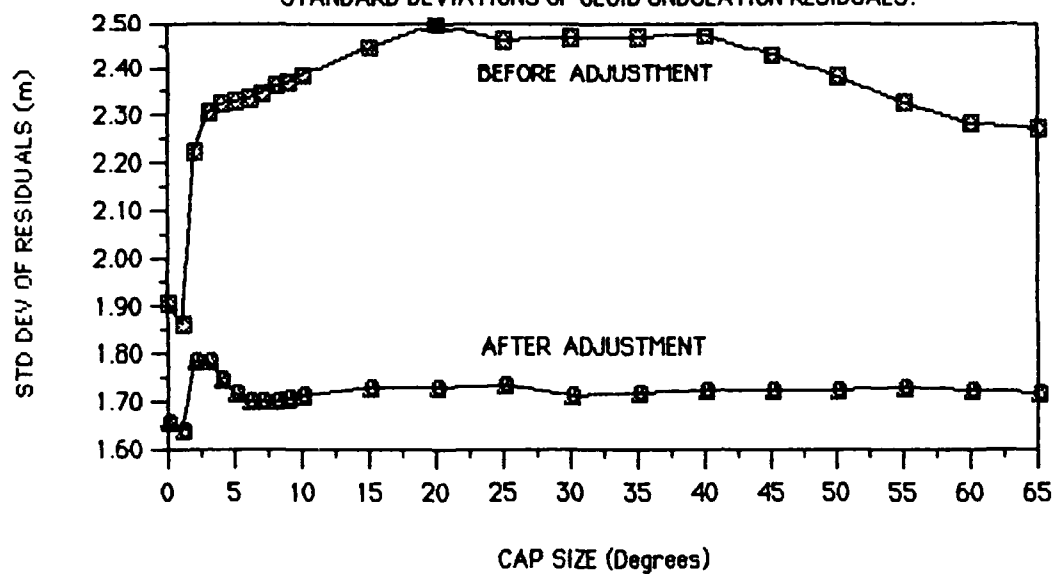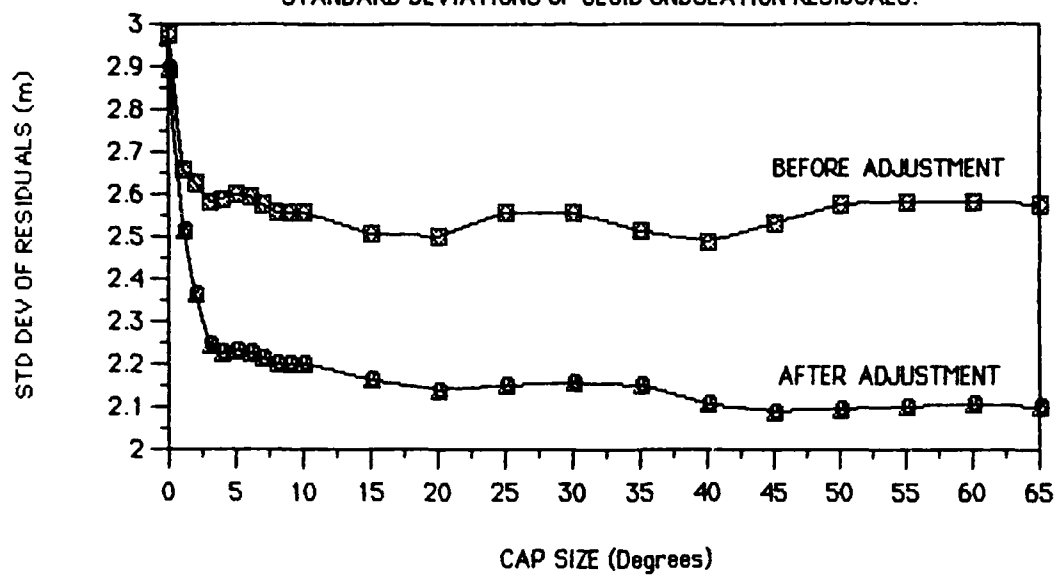FIG 5. ARCTIC.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

28
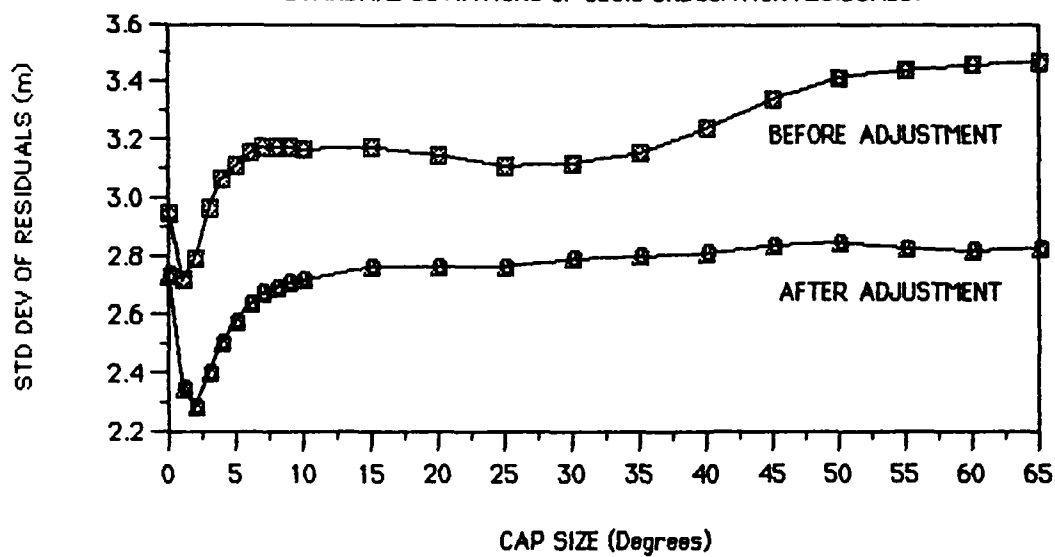
FIG 6. ATLANTIC.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.
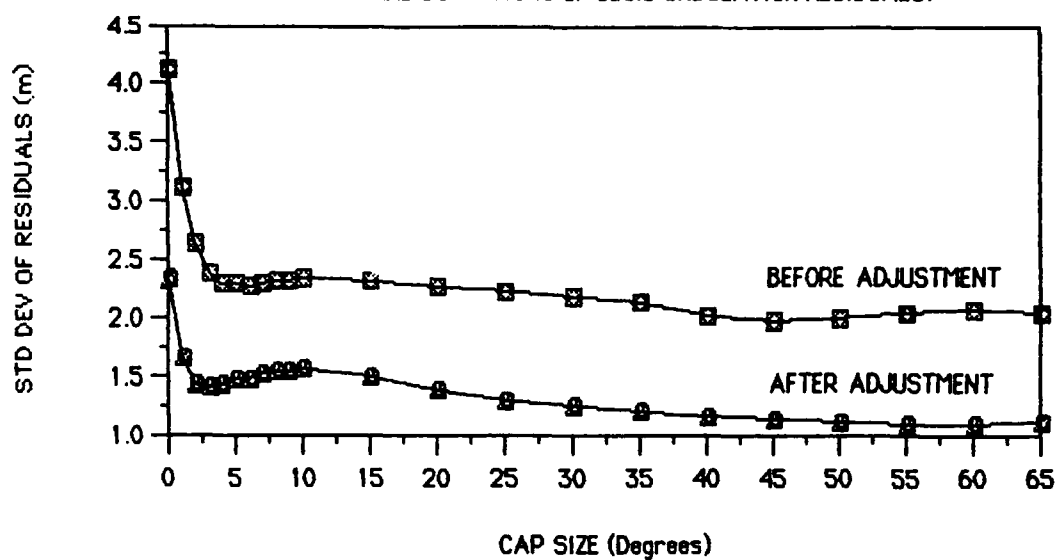
FIG 7. AUSTRALIA.
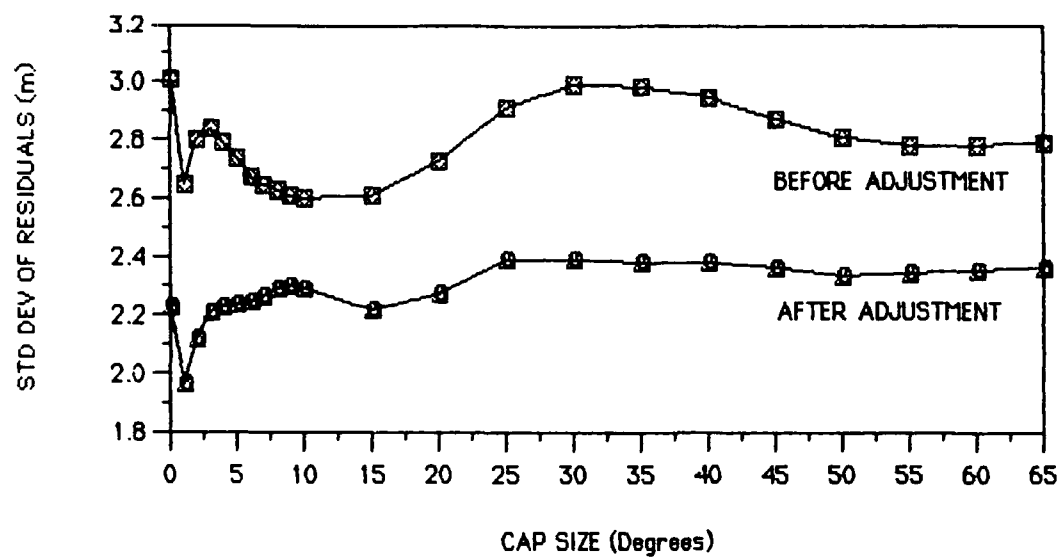STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.
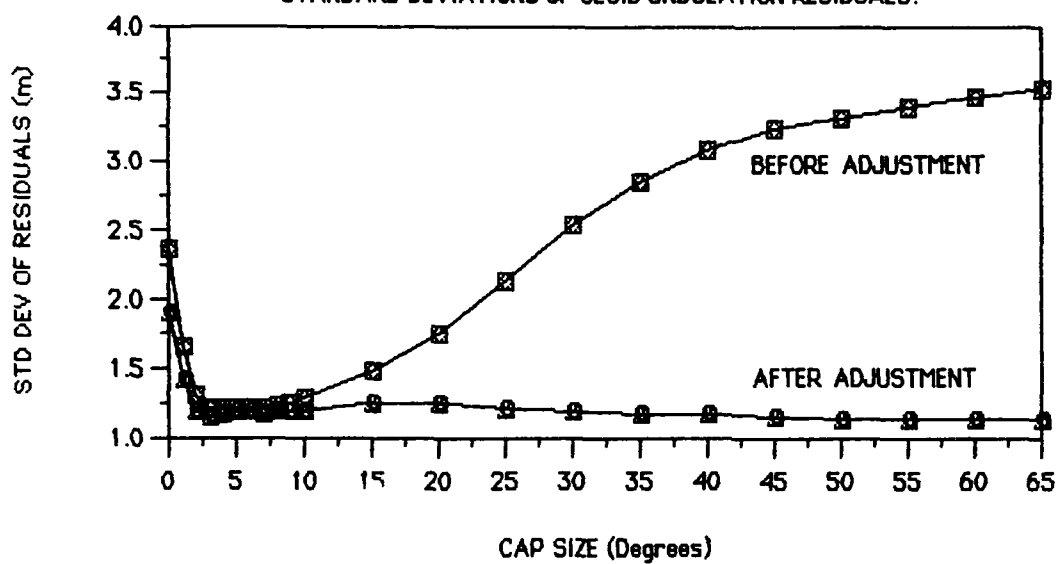
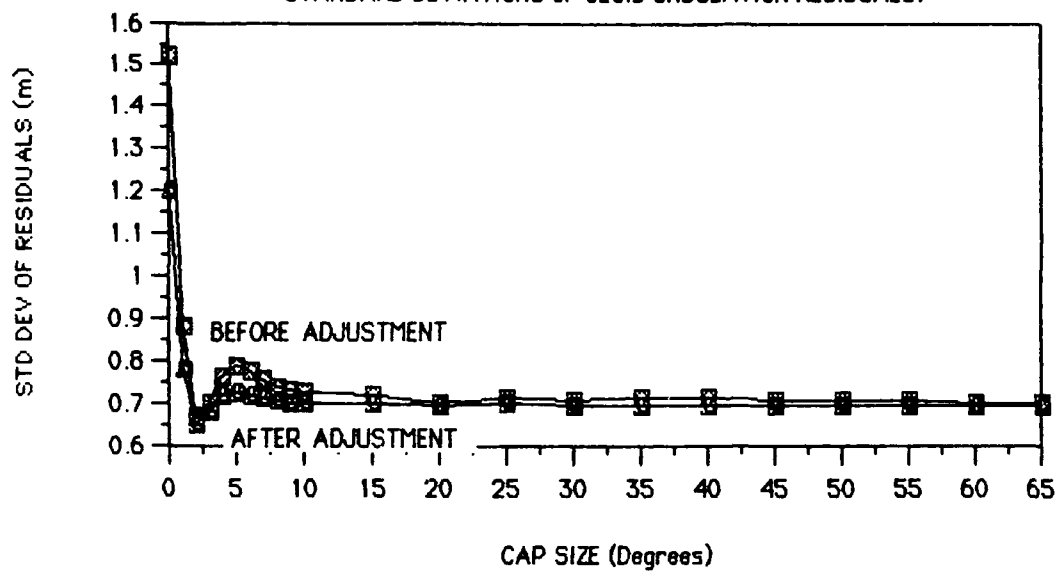FIG 8. EAST COAST OF SOUTH AMERICA.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.
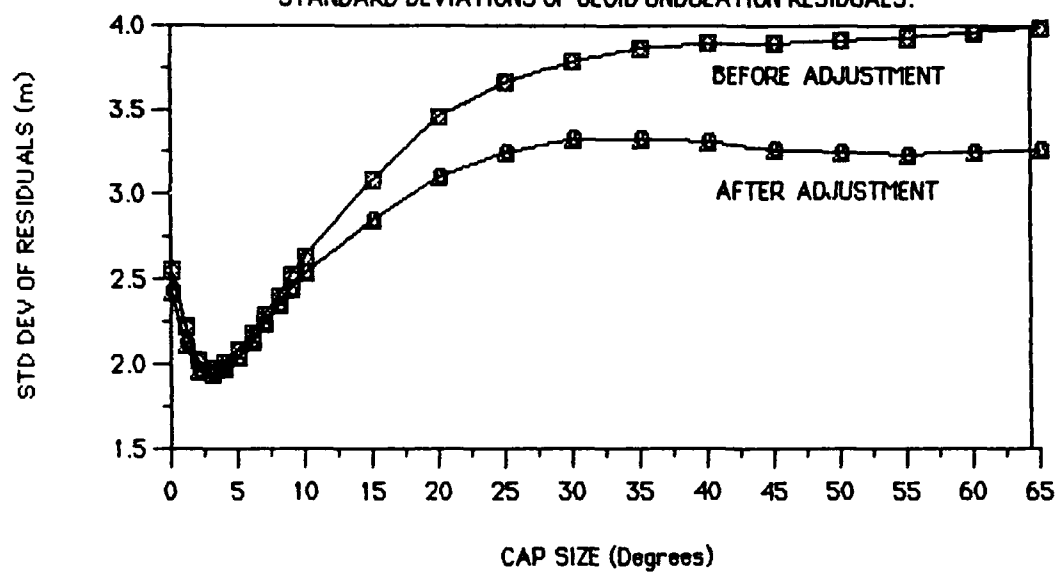
FIG 9. EAST COAST OF U.S. – CANADA.
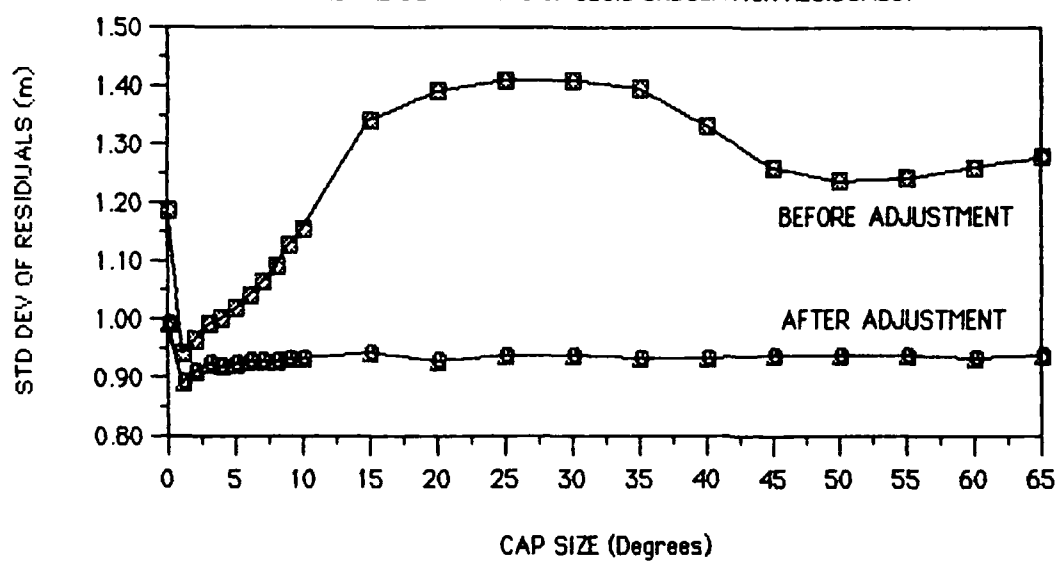STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.
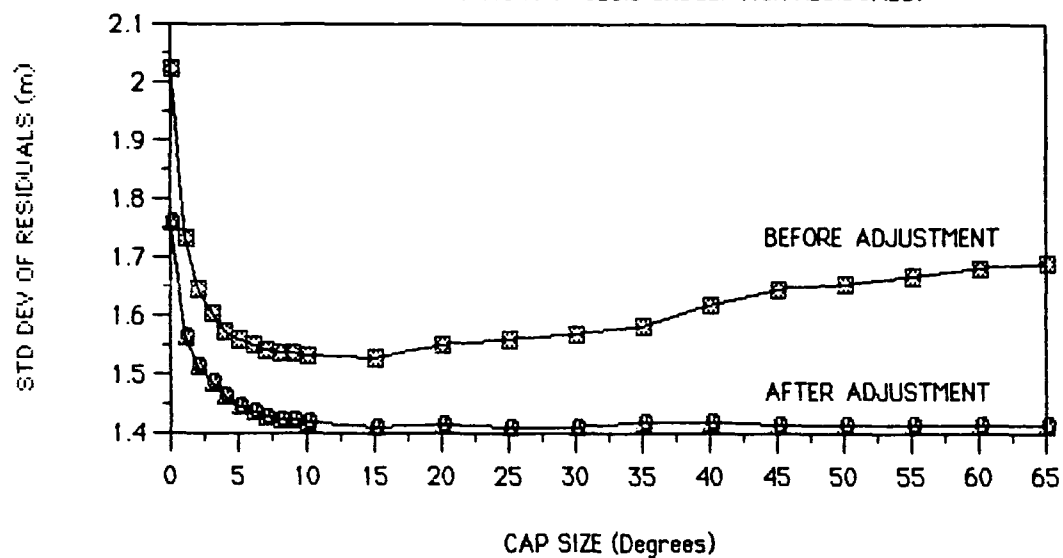
FIG 10. EUROPE.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

BEFORE ADJUSTMENT

AFTER ADJUSTMENT

CAP SIZE (Degrees)

STD DEV OF RESIDUALS (m)

33

FIG 11. MEXICO – CENTRAL AMERICA.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

FIG 12. NORTH PACIFIC.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

FIG 13. NORTHWESTERN AFRICA.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

FIG 14. SOUTH PACIFIC.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

FIG 15. U.S. GULF STATES.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

FIG 16. WEST COAST OF U.S. – CANADA.
STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

FIG 17. WEST COAST OF SOUTH AMERICA.
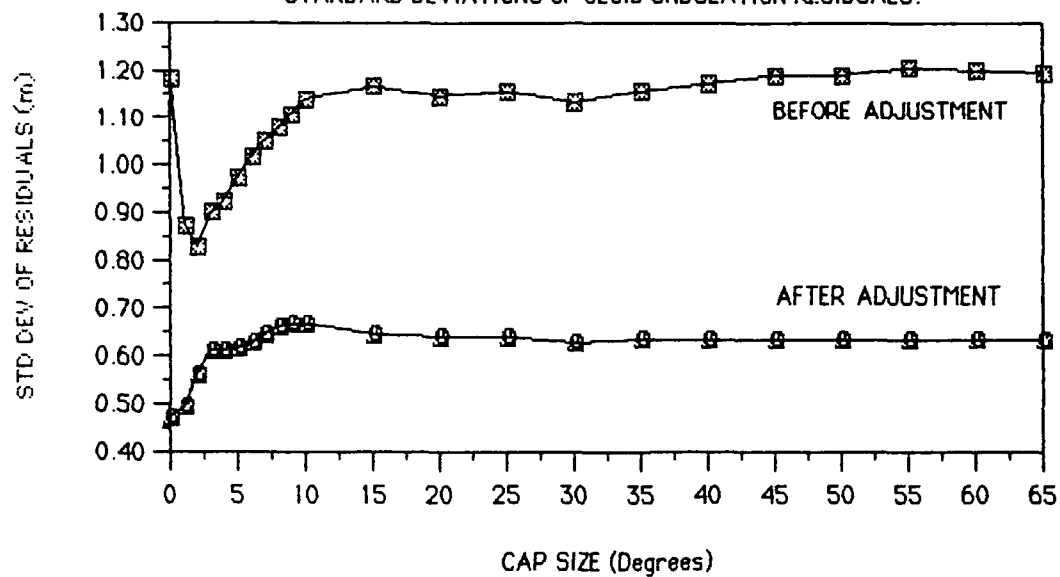STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

FIG 18. WEST INDIES.
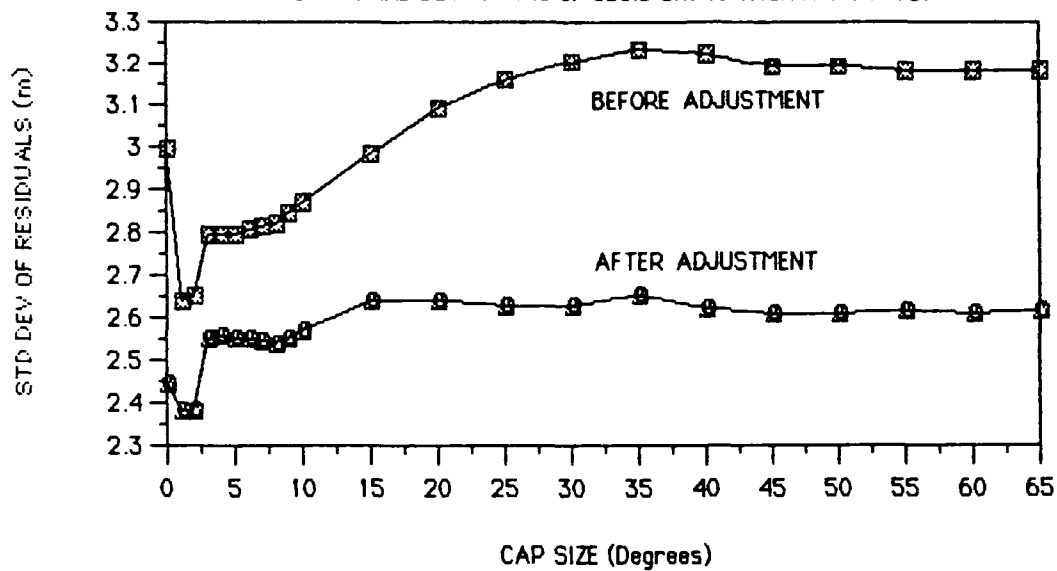STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

FIG 19. EASTERN HEMISPHERE.
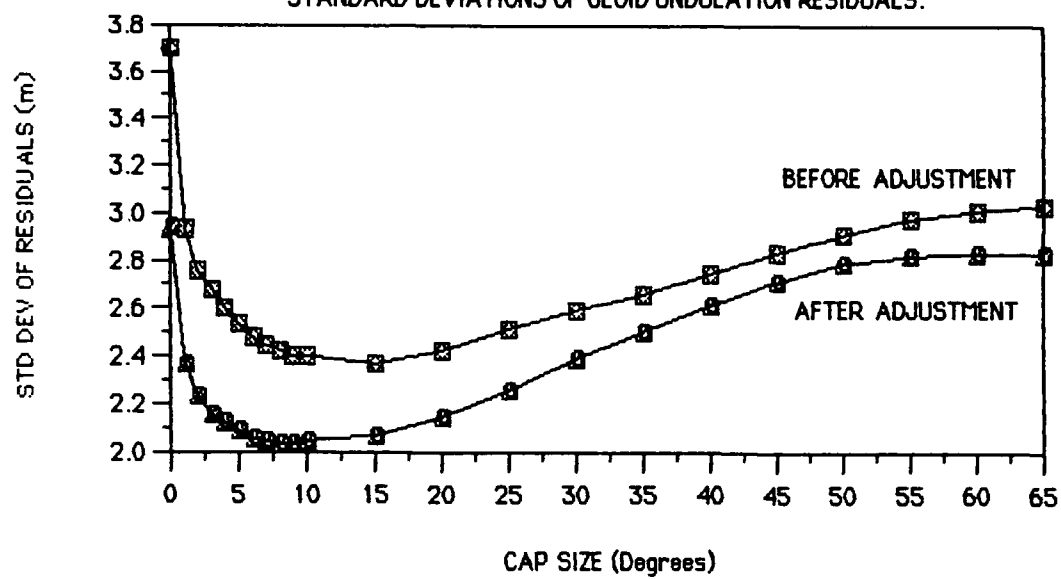STANDARD DEVIATIONS OF GEOID UNDULATION RESIDUALS.

FIG 20. WESTERN HEMISPHERE.
STANDARD DEVIATION OF THE GEOID UNDULATION RESIDUALS.